Math 60 9.4 Simplifying Radical Expressions Using Properties of Radicals

Objectives 1) Use the product property to
- multiply radical expressions
- simplify radical expressions.

2) Use the quotient property to simplify radical expressions.

Both the product and quotient properties depend on the following:

IF If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers

↳ This means that these properties are not true for $\sqrt[n]{a}$ or $\sqrt[n]{b}$ not real.

↳ In 9.9, we'll learn another method for non-real radicals.

↳ This means either:
1) n is odd
OR
2) if n is even, a and b are non-negative

But if $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers

then:

product property of radicals

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

quotient property of radicals

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Both of these properties can be used in two directions

$\Rightarrow$ start with two radicals, combine to one

$\Leftarrow$ start with one radical, split into two

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Multiply.

① $\sqrt{7} \cdot \sqrt{2}$

$= \sqrt{7 \cdot 2}$

$= \boxed{\sqrt{14}}$

$\sqrt{7}$ and $\sqrt{2}$ are real numbers

multiply radicands

② $\sqrt[3]{3} \cdot \sqrt[3]{11}$

$= \sqrt[3]{3 \cdot 11}$

$= \boxed{\sqrt[3]{33}}$

$\sqrt[3]{3}$ and $\sqrt[3]{11}$ are real numbers

multiply radicands

③ $\sqrt{n+4} \cdot \sqrt{n-4}$

$= \sqrt{(n+4)(n-4)}$

$= \boxed{\sqrt{n^2-16}}$

Assume $n \geq 4$.
$\sqrt{n+4}$, $\sqrt{n-4}$ are real numbers so long as $n \geq 4$.

multiply radicands, using parentheses

FOIL

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Assume all variables are real numbers

skip (4) $\sqrt[5]{7a^3} \cdot \sqrt[5]{3a}$

$\sqrt[5]{7a^3}$ and $\sqrt[5]{3a}$ are real numbers for all real values of a.

$= \sqrt[5]{7a^3 \cdot 3a}$ multiply radicands

$= \boxed{\sqrt[5]{21a^4}}$

(5) $\sqrt{-2} \cdot \sqrt{-3}$

$\sqrt{-2}$ and $\sqrt{-3}$ are not real numbers

= $\boxed{\text{cannot be multiplied}}$

We'll learn another way to do this question in section 9.9.

(6) Simplify $\sqrt{25-16}$

$= \sqrt{9}$ order of operations: simplify inside first.

$= \boxed{3}$

(7) Simplify $\sqrt{25} - \sqrt{16}$

$= 5 - 4$

$= \boxed{1}$

Notice: $\sqrt{25-16} \neq \sqrt{25} - \sqrt{16}$

We cannot split radicals apart when the radicand is added or subtracted.

The product property can be used in the opposite direction also:

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

We will use this to simplify (lower) the radicand.

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(8) Simplify $\sqrt{24}$

GOAL: Find two factors of 24 where one is a perfect square, because this is a square root, index 2.

$24 = 1 \times 24$ no

2×12 no

3×8 8 is a perfect cube, no

$\underline{4} \times 6$ 4 is a perfect square, yes

$\sqrt{24} = \sqrt{4 \cdot 6}$ rewrite 24 using a perfect square factor

$= \sqrt{4} \cdot \sqrt{6}$ split apart the radicand

$= 2 \cdot \sqrt{6}$ simplify $\sqrt{\text{perfect square}}$

$= \boxed{2\sqrt{6}}$ remove multiply symbol.

(9) Simplify $\sqrt[3]{24}$

GOAL: Find two factors of 24 where one is a perfect cube, because this is a cube root, index 3.

$24 = 1 \times 24$ no

2×12 no

3×8 yes b/c 8 is a perfect cube

4×6 no

$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3}$ rewrite 24 using a perfect cube factor

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$ split apart radical

$= \boxed{2\sqrt[3]{3}}$ Simplify $\sqrt[3]{\text{perfect cube}}$

Note: It is helpful to write the perfect factor first, but not required.

$\sqrt{24} = \sqrt{6 \cdot 4} = \sqrt{6} \cdot \sqrt{4} = \sqrt{6} \cdot 2 = 2\sqrt{6}$

$\sqrt[3]{24} = \sqrt[3]{3 \cdot 8} = \sqrt[3]{3} \cdot \sqrt[3]{8} = \sqrt[3]{3} \cdot 2 = 2\sqrt[3]{3}$.

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We want to get a quicker way (and a deeper understanding) of the relationship between an exponent inside a radical and the index of the radical.

$$\underbrace{\sqrt{3}\cdot\sqrt{3}} = \sqrt{3\cdot 3} = \sqrt{3^2} = \sqrt{9} = 3$$

it takes 2 radicals to simplify index 2 radical.

$$\underbrace{\sqrt{3}\cdot\sqrt{3}}\cdot\sqrt{3} = \sqrt{3\cdot3\cdot3} = \sqrt{27} = \sqrt{9\cdot 3} = \sqrt{9}\cdot\sqrt{3} = 3\sqrt{3}$$

This is leftover.

These two combine and simplify

$$\underbrace{\sqrt{3}\cdot\sqrt{3}}_{\text{combine}}\cdot\underbrace{\sqrt{3}\cdot\sqrt{3}}_{\text{combine}} = \sqrt{3^4} = 3^{4/2} = 3^2 = 9$$

$$\underbrace{\sqrt{3}\cdot\sqrt{3}}_{\text{combine}}\cdot\underbrace{\sqrt{3}\cdot\sqrt{3}}_{\text{combine}}\cdot\underbrace{\sqrt{3}}_{\text{leftover}} = \sqrt{3^5} = \sqrt{3^4\cdot 3} = 3^{4/2}\cdot\sqrt{3} = 3^2\sqrt{3} = 9\sqrt{3}$$

divide exponent by index 2.

Cube roots:

$$\sqrt[3]{2}\cdot\sqrt[3]{2} = \sqrt[3]{4} \quad \text{does not simplify!}$$

$$\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}} = \sqrt[3]{2\cdot2\cdot2} = \sqrt[3]{2^3} = \sqrt[3]{8} = 2$$

These 3 combine and simplify

$$\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}_{\text{combine}}\cdot\sqrt[3]{2} = \sqrt[3]{2^4} = \sqrt[3]{2^3}\cdot\sqrt[3]{2} = 2\sqrt[3]{2}$$

leftover

$$\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}_{\text{combine}}\cdot\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}}_{\text{left}} = \sqrt[3]{2^5} = \sqrt[3]{2^3\cdot 2^2} = 2\sqrt[3]{4}$$

$$\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}_{\text{combine}}\cdot\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}_{\text{combine}} = \sqrt[3]{2^6} = 2^{6/3} = 2^2 = 4$$

$$\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}\cdot\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}\cdot\underbrace{\sqrt[3]{2}}_{\text{leftover}} = \sqrt[3]{2^7} = \sqrt[3]{2^6\cdot 2} = 4\sqrt[3]{2}$$

$$\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}\cdot\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}\cdot\sqrt[3]{2}}\cdot\underbrace{\sqrt[3]{2}\cdot\sqrt[3]{2}}_{\text{leftover}} = \sqrt[3]{2^8} = \sqrt[3]{2^6\cdot 2^2} = 4\sqrt[3]{4}$$

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The difficulty with the "perfect squares" or "perfect cubes" approach is that you have to know arithmetic, sometimes with large numbers, really well.

So we want another approach, based on exponents.

		exp inside ÷ index = exp result
Notice:	$\sqrt{2^2} = \sqrt{4} = 2 = 2^1$	$2 \div 2 = 1$
	$\sqrt{2^4} = \sqrt{16} = 4 = 2^2$	$4 \div 2 = 2$
	$\sqrt{2^6} = \sqrt{64} = 8 = 2^3$	$6 \div 2 = 3$
	$\sqrt{2^8} = \sqrt{256} = 16 = 2^4$	$8 \div 2 = 4$
similarly:	$\sqrt{3^2} = \sqrt{9} = 3 = 3^1$	$2 \div 2 = 1$
	$\sqrt[3]{2^3} = \sqrt[3]{8} = 2 = 2^1$	$3 \div 3 = 1$
	$\sqrt[3]{2^6} = \sqrt[3]{64} = 4 = 2^2$	$6 \div 3 = 2$
	$\sqrt[4]{2^8} = \sqrt[4]{256} = 4 = 2^2$	$8 \div 4 = 2.$

DIVIDE EXPONENT BY INDEX

Since dividing by the index simplifies only when the exponent divides evenly, we want to break up our radicand into bases with exponents that are the biggest exponents that will divide by the index.

(10) $\sqrt{2^3} = \sqrt{2^{2+1}} = \sqrt{2^2 \cdot 2^1} = \sqrt{2^2} \cdot \sqrt{2} = \boxed{2\sqrt{2}}$

(11) $\sqrt{3^5} = \sqrt{3^{4+1}} = \sqrt{3^4 \cdot 3^1} = \sqrt{3^4} \cdot \sqrt{3} = 3^2\sqrt{3} = \boxed{9\sqrt{3}}$

(12) $\sqrt[3]{2^5} = \sqrt[3]{2^{3+2}} = \sqrt[3]{2^3 \cdot 2^2} = \sqrt[3]{2^3} \cdot \sqrt[3]{2^2} = \boxed{2\sqrt[3]{4}}$

(13) $\sqrt[3]{2^4} = \sqrt[3]{2^{3+1}} = \sqrt[3]{2^3 \cdot 2^1} = \sqrt[3]{2^3} \cdot \sqrt[3]{2} = \boxed{2\sqrt[3]{2}}$

(14) $\sqrt[3]{2^{10}} = \sqrt[3]{2^{9+1}} = \sqrt[3]{2^9 \cdot 2^1} = \sqrt[3]{2^9} \cdot \sqrt[3]{2} = 2^{9/3}\sqrt[3]{2} = 2^3\sqrt[3]{2}$
$= \boxed{8\sqrt[3]{2}}$

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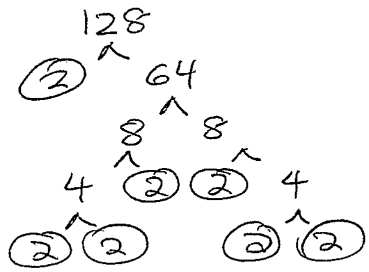
15 Simplify $2\sqrt[3]{128}$

$\boxed{\text{option 1}}$ Use exponents



step 1: find prime factorization

$128 = 2^7$

means

$= 2\sqrt[3]{2^7}$

step 2: Use exponent laws in reverse to split 2^7 into the biggest exponent that's divisible by the index 3

$$2^7 = 2^6 \cdot 2$$
$$= \cancel{2^5 \cdot 2^2}$$
$$= \cancel{2^4 \cdot 2^3}$$

exp 6 is divisible by 3

neither 5 nor 2 is divisible by 3.

exp 3 is divisible by 3 but 4 > 3 means we'd have to simplify again. It's not the biggest exp.

$= 2\sqrt[3]{2^6 \cdot 2}$

step 3: Use product property to split radical into two.

$= 2\sqrt[3]{2^6} \cdot \sqrt[3]{2}$

step 4: simplify radical by dividing exponent by index.

$= 2 \cdot 2^{6/3} \cdot \sqrt[3]{2}$

step 5: simplify exponent and do arithmetic

$= 2 \cdot 2^2 \cdot \sqrt[3]{2}$

$= 2 \cdot 4 \cdot \sqrt[3]{2}$

$= \boxed{8\sqrt[3]{2}}$

$\boxed{\text{option 2}}$ Find a perfect cube factor of 128

$= 2\sqrt[3]{64 \cdot 2}$

$= 2\sqrt[3]{64} \cdot \sqrt[3]{2} = 2 \cdot 4 \cdot \sqrt[3]{2} = \boxed{8\sqrt[3]{2}}$

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16 Simplify $\sqrt{128x^2}$ Assume variables can be any real #.

option 1: Use exponents. See 15 for prime factorization

$= \sqrt{2^7 \cdot x^2}$

want the biggest exponent that's divisible by index 2.

$2^7 = 2^6 \cdot 2$ $\quad 6 \div 2$ ✓

$\cancel{= 2^5 \cdot 2^2}$ $\quad 2 \div 2$ but not biggest

$\cancel{= 2^4 \cdot 2^3}$ $\quad 4 \div 2$ but not biggest

$= \sqrt{2^6 \cdot 2 \cdot x^2}$

$= \sqrt{2^6} \cdot \sqrt{2} = \sqrt{x^2}$ product property

$= 2^{6/2} \cdot \sqrt{2} \cdot |x|$ divide exponent by index

$\sqrt{x^2} = |x|$ when x could be negative

$= 2^3 \sqrt{2} \cdot |x|$

$= \boxed{8\sqrt{2} \cdot |x|}$ be sure $|x|$ is not inside radical!

$= \boxed{8|x| \cdot \sqrt{2}}$

option 2: want a perfect square factor.

$= \sqrt{64 \cdot 2 \cdot x^2}$

$= \sqrt{64}\ \sqrt{2}\ \sqrt{x^2}$

$= \boxed{8\sqrt{2}\ |x|}$

$= \boxed{8\,|x|\,\sqrt{2}}$

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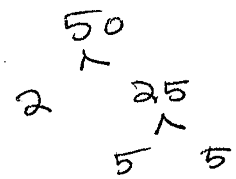
skip. (17) $\sqrt[4]{50}$

4th root $\Rightarrow$ want perfect 4th power



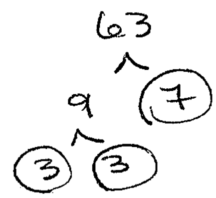
$50 = 2 \cdot \underbrace{5 \cdot 5}_{\text{only 2, not 4.}}$

= $\boxed{\text{cannot be simplified further}}$ ← not now, not ever.

(18) Simplify $\dfrac{6 - \sqrt{63}}{3}$

index 2 $\Rightarrow$ want perfect square factor

$63 = \underbrace{3 \cdot 3}_{\substack{9 \\ \text{square}}} \cdot 7$



$= \dfrac{6 - \sqrt{9 \cdot 7}}{3}$ rewrite radicand

$= \dfrac{6 - \sqrt{9} \cdot \sqrt{7}}{3}$ split radical + simplify perfect square

$= \dfrac{6}{3} - \dfrac{3\sqrt{7}}{3}$ ← monomial division — divide each term and reduce fractions $\frac{6}{3}$ and $\frac{3}{3}$

$= \boxed{2 - \sqrt{7}}$

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19 Simplify $\sqrt{32y^8}$. Assume $y \geq 0$.

This means $\sqrt{y}$ is a real number, and $\sqrt{y^2} = y$ w/o absolute values.

Method 1: (preferred)

$= \sqrt{2^5 \cdot y^8}$ write with exponents.

$= \sqrt{2^4 \cdot 2 \cdot y^8}$ $2^{4+1} = 2^5 = 2^4 \cdot 2$ find largest exponent (4) that is divisible by index (2)

$= \sqrt{2^4} \cdot \sqrt{2} \cdot \sqrt{y^8}$ product property

$= 2^{4/2} \sqrt{2}\, y^{8/2}$ divide exponent by index

$= 2^2 \sqrt{2}\, y^4$ simplify

$= 4\sqrt{2}\, y^4$

$= \boxed{4y^4\sqrt{2}}$ write radical last

Method 2: longer

$= \sqrt{\underbrace{2 \cdot 2} \cdot \underbrace{2 \cdot 2} \cdot 2 \cdot \underbrace{y \cdot y} \cdot \underbrace{y \cdot y} \cdot \underbrace{y \cdot y} \cdot \underbrace{y \cdot y}}$

leftover

each pair simplifies
$\sqrt{2} \cdot \sqrt{2} = 2$
$\sqrt{y} \cdot \sqrt{y} = y$

$= 2 \cdot 2 \cdot \sqrt{2} \cdot y \cdot y \cdot y \cdot y$

$= 2^2 \cdot y^4 \sqrt{2}$

$= \boxed{4y^4\sqrt{2}}$

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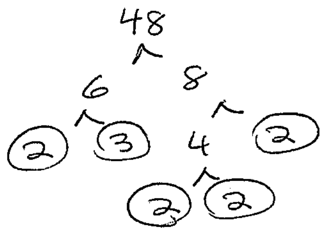
Simplify. Assume variables are

(20) $\sqrt{48x^3}$



$= \sqrt{2^4 \cdot 3 \cdot x^3}$

exp already divisible by index 2

leftover

split $x^{2+1} = x^2 \cdot x$ to get an exponent divisible by index 2.

$= \sqrt{2^4 \cdot 3 \cdot x^2 \cdot x}$

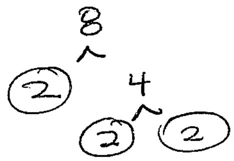
$= \sqrt{2^4} \cdot \sqrt{3} \cdot \sqrt{x^2} \cdot \sqrt{x}$ product property

$= 2^{4/2} \sqrt{3}\ x^{2/2} \cdot \sqrt{x}$ divide exponent by index

$= 2^2 \cdot \sqrt{3} \cdot x \cdot \sqrt{x}$ combine $\sqrt{3} \cdot \sqrt{x}$ using product property

$= \boxed{4x\sqrt{3x}}$

(21) $\sqrt[3]{8a^5b^{10}}$



$= \sqrt[3]{2^3 \cdot a^3 \cdot a^2 \cdot b^9 \cdot b}$

largest exp divisible by index 3

largest exp divisible by index 3

$= 2^{3/3} \cdot a^{3/3} \cdot \sqrt[3]{a^2} \cdot b^{9/3}\ \sqrt[3]{b}$ divide exp by index

$= \boxed{2ab^3\sqrt[3]{a^2b}}$ combine $\sqrt[3]{a^2}$ and $\sqrt[3]{b}$ using product property

(22) $2\sqrt[3]{9x^2} \cdot \sqrt[3]{3x^2}$ ← Neither radical can be simplified. Re-combine and then split differently

$= 2\sqrt[3]{9 \cdot 3 \cdot x^2 \cdot x^2}$

$= 2\sqrt[3]{27x^4}$

$= 2\sqrt[3]{27} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x}$ largest exp divisible by index 3

cont →

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continued

$= 2 \cdot 3 \cdot x \cdot \sqrt[3]{x}$

$= \boxed{6x\sqrt[3]{x}}$

skip 23 $\sqrt{\dfrac{24}{49}}$

$= \dfrac{\sqrt{24}}{\sqrt{49}}$ quotient property

$= \dfrac{\sqrt{4}\cdot\sqrt{6}}{7}$ ← product property

$= \boxed{\dfrac{2\sqrt{6}}{7}}$

24 $\sqrt[3]{\dfrac{5p}{27}}$

$= \dfrac{\sqrt[3]{5p}}{\sqrt[3]{27}}$ quotient property

$= \boxed{\dfrac{\sqrt[3]{5p}}{3}}$ cube root → perfect cube $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

25 $\sqrt[4]{\dfrac{5y^3}{16z^4}}$

$= \dfrac{\sqrt[4]{5y^3}}{\sqrt[4]{16z^4}}$ quotient property

$= \dfrac{\sqrt[4]{5y^3}}{2z}$

$\left.\begin{array}{l}\sqrt[4]{16} = \sqrt[4]{2^4} = 2 \\ \sqrt[4]{z^4} = z\end{array}\right\}$ index 4 ⇒ need perfect 4th powers.

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(26) $\dfrac{\sqrt{45y^5}}{\sqrt{5y}}$

option 1: Use the quotient property to re-combine.

$= \sqrt{\dfrac{45y^5}{5y}}$

$= \sqrt{9y^4}$ simplify $\frac{45}{5}$ and $\frac{y^5}{y} = y^{5-1} = y^4$

$= \sqrt{9}\sqrt{y^4}$ product property

$= \boxed{3y^2}$

option 2: Simplify first

$= \dfrac{\sqrt{9 \cdot 5 \cdot y^4 \cdot y}}{\sqrt{5y}}$

$= \dfrac{3y^2 \cancel{\sqrt{5y}}}{\cancel{\sqrt{5y}}}$ cancel common factors

$= \boxed{3y^2}$

(27) $\dfrac{-2\sqrt[3]{250n}}{\sqrt[3]{2n^4}}$

option 1: Use the quotient property to re-combine.

$= -2 \cdot \sqrt[3]{\dfrac{250n}{2n^4}}$

$= -2\sqrt[3]{\dfrac{125}{n^3}}$ $\frac{250}{2} = 125$ $\frac{n}{n^4} = \frac{1}{n^3}$

continued

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continued

$$= -2\,\frac{\sqrt[3]{125}}{\sqrt[3]{n^3}}$$ quotient property to split apart.

$$= \frac{-2\cdot 5}{n}$$

$$= \boxed{\frac{-10}{n}}$$

option 2: Simplify first.

$$= \frac{-2\sqrt[3]{2\cdot 5^3\cdot n}}{\sqrt[3]{2n^3\cdot n}}$$

250
10 25
2 5 5 5

$$= \frac{-2\,\cancel{\sqrt[3]{2}}\cdot 5\cdot \cancel{\sqrt[3]{n}}}{\cancel{\sqrt[3]{2}}\cdot n\cdot \cancel{\sqrt[3]{n}}}$$

cancel common factors

$$= \boxed{\frac{-10}{n}}$$

(28) $$\frac{\sqrt[3]{320x^4y^2}}{\sqrt[3]{-5x^{-2}y^5}}$$

option 1: re-combine using quotient property

$$= \sqrt[3]{\frac{320}{-5}\cdot\frac{x^4}{x^{-2}}\cdot\frac{y^2}{y^5}}$$

simplify $\frac{320}{-5} = -64$

$$\frac{x^4}{x^{-2}} = x^{4-(-2)} = x^{4+2} = x^6$$

$$= \sqrt[3]{\frac{-64x^6}{y^3}}$$

$$\frac{y^2}{y^5} = \frac{1}{y^{5-2}} = \frac{1}{y^3}$$

continued

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continued

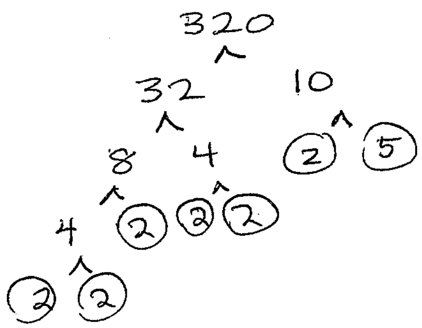
$$= \frac{\sqrt[3]{-64} \cdot \sqrt[3]{x^6}}{\sqrt[3]{y^3}}$$ product and quotient properties to split apart

$$= \frac{-4\,x^{6/3}}{y}$$ divide exp by index.

$$= \boxed{\frac{-4x^2}{y}}$$

option 2: simplify first.

$$= \frac{\sqrt[3]{2^6 \cdot 5 \cdot x^3 \cdot x \cdot y^2}}{\sqrt[3]{-5 \cdot x^{-2} \cdot y^3 \cdot y^2}}$$



$$= \frac{\sqrt[3]{2^6} \cdot \sqrt[3]{5} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x} \cdot \sqrt[3]{y^2}}{\sqrt[3]{-5}\ \sqrt[3]{x^{-2}} \cdot \sqrt[3]{y^3} \cdot \sqrt[3]{y^2}}$$ product property to split apart

$$= \frac{2^{6/3} \cdot \cancel{\sqrt[3]{5}} \cdot x \cdot \sqrt[3]{x} \cdot \cancel{\sqrt[3]{y^2}}}{-\cancel{\sqrt[3]{5}} \cdot \sqrt[3]{x^{-2}} \cdot y \cdot \cancel{\sqrt[3]{y^2}}}$$ simplify, divide exp by index

cancel common factors

$$= \frac{2^2 \cdot x \cdot \sqrt[3]{x}}{-\sqrt[3]{x^{-2}} \cdot y}$$

move negative exp to numerator to make it positive

move negative coefficient to numerator

$$= \frac{-4x\,\sqrt[3]{x} \cdot \sqrt[3]{x^2}}{y}$$

$$= \frac{-4x\,\sqrt[3]{x^3}}{y}$$ product property

$$= \frac{-4x \cdot x}{y} = \boxed{\frac{-4x^2}{y}}$$